On logic in complexity theory, and birds and frogs

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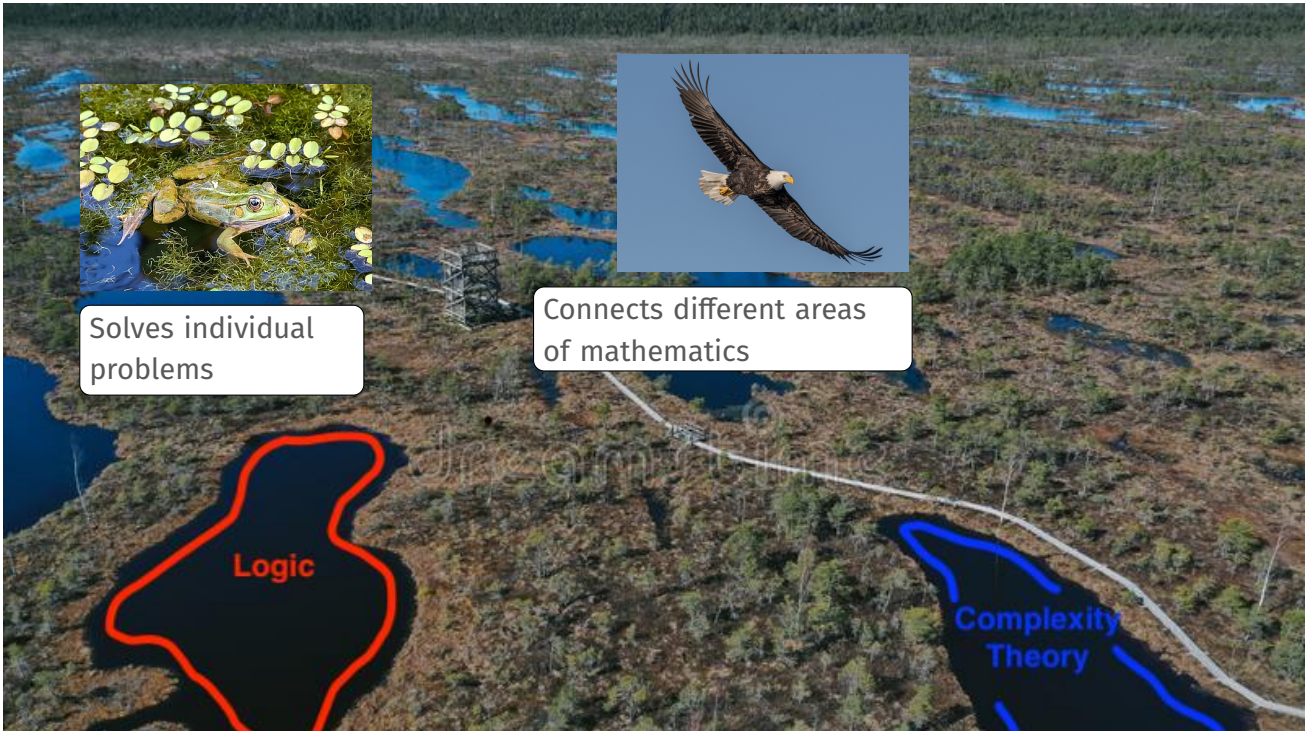
Birds and Frogs

AMS Einstein Lecture 2008

Freeman Dyson

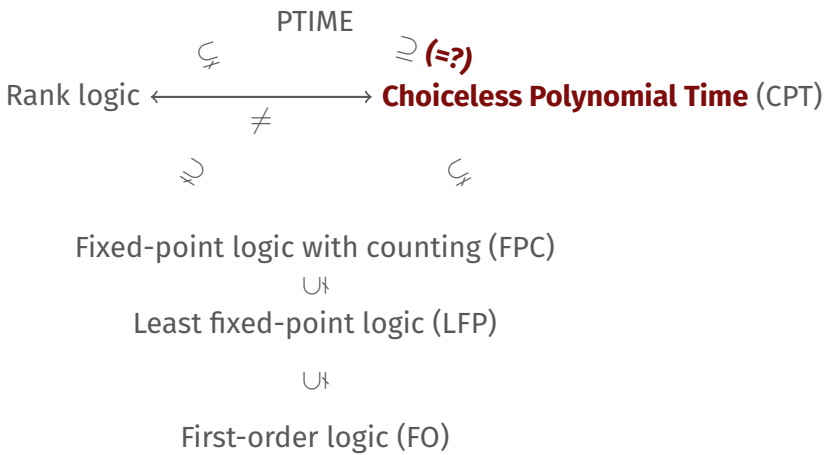
Birds and frogs in mathematics



Is there a logic capturing PTIME?

- **Logics** are symmetry-invariant computation models.
- A logic $\mathcal{L}$ **captures** a **complexity class** $\mathcal{C}$ if the evaluation problem for $\mathcal{L}$-sentences is in $\mathcal{C}$ and if every class of finite structures decidable in $\mathcal{C}$ is definable by an $\mathcal{L}$-sentence.
- Central question in finite model theory: Does there exist a logic that captures PTIME?
- If not, then P $\neq$ NP (because existential second-order logic captures NP).
- Proving lower bounds against logics is easier than for general algorithms.

Landscape of logics contained in PTIME



Choiceless Polynomial Time

CPT is a “functional programming language” with hereditarily finite sets as data structures.
A CPT-program is built out of set-theoretic terms, such as:

- $\texttt{Pair}(a, b) := \{a, b\}$.
- $\texttt{Union}(a) := \bigcup a$.
- Comprehension: $\{t \ : \ x \in a \ : \ \varphi\} := \{t(x) \mid x \in a, \mathfrak{A} \models \varphi(x)\}$.
- Iteration: Terms can be iteratively applied to their own output.

Limitations of CPT:

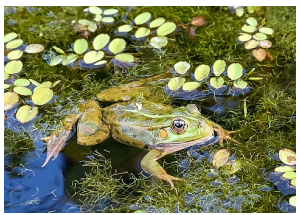
- Length of iterations and size of constructed sets *polynomially bounded* in input size $|\mathfrak{A}|$.
- The computed sets are *symmetric* under the automorphisms of $\mathfrak{A}$.
- **Goal:** Prove *inexpressibility* results for CPT.

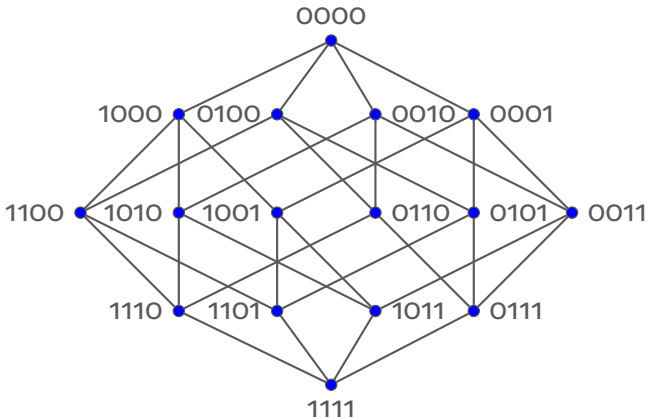
Symmetry-invariance of hereditarily finite sets

$\mathfrak{A}$:

- $\{\{a, b, c\}, \{d, e, f\}\}$ is symmetric.
- $\{a, d\}$ is *not* symmetric.

Lower bounds for CPT via analysis of highly symmetric structures

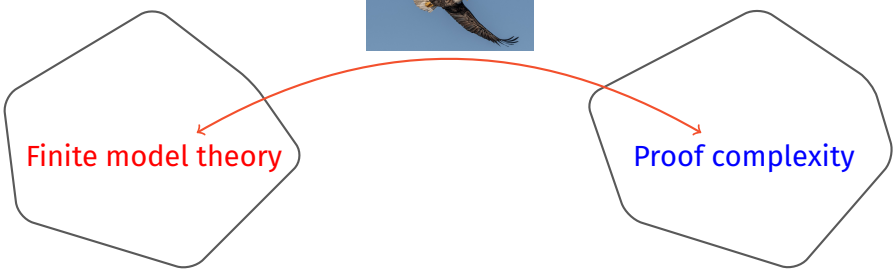




The 4-dimensional hypercube.

Theorem (P., CSL 2021)
No CPT-program can define an ordered partition of the n-dimensional hypercube into parts of at most logarithmic size.

Another approach: Lower bounds for CPT via propositional proof complexity



Theorem (P., CSL 2023)
If CPT can distinguish all pairs of non-isomorphic graphs in a graph class $\mathcal{K}$, then so can the degree-3 extended polynomial calculus (EPC_3).

Consequence: Lower bounds for the graph isomorphism problem in EPC_3 transfer to CPT.

Part II: PhD impressions

Finding the right position

- Check out advertised positions on mailing lists such as **DMANET** or the **finite-model-theory list**.
- There are generally more open positions than applicants.
- Choose a **topic** you personally find **exciting**.
- Other aspects to consider: is the supervisor active in research, how big is the group, how much teaching and other tasks does the job involve?

Pros and Cons of being a theoretical researcher

Pros	Cons
self-dependance	self-dependance
freedom	less money than in industry
extremely challenging	lack of relevance
varying topics	
teaching	
interesting people	